

Math A and I HL Year 2 Summer Assignment - No Stats [46 marks]

Review of Year 1 Topics

Give these problems a shot toward the end of summer. These problems are representative of what we did last year. This should take you about one hour. The answers are included at the end.

1a. [1 mark] The Osaka Tigers basketball team play in a multilevel stadium. The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

Write down the value of the common difference, d



1b. [2 marks] Calculate the price of a ticket in the 16th row.

1c. [3 marks] Find the total cost of buying 2 tickets in each of the first 16 rows.

2a. [2 marks] The intensity level of sound, L measured in decibels (dB), is a function of the sound intensity, S watts per square metre (W m^{-2}). The intensity level is given by the following formula.
 $L = 10 \log_{10}(S \times 10^{12})$, $S \geq 0$. An orchestra has a sound intensity of $6.4 \times 10^{-3} \text{ W m}^{-2}$. Calculate the intensity level, L of the orchestra.

2b. [2 marks] A rock concert has an intensity level of 112 dB. Find the sound intensity, S .

3a. [2 marks] Give your answers to this question correct to two decimal places. Gen invests \$2400 in a savings account that pays interest at a rate of 4% per year, compounded annually. She leaves the money in her account for 10 years, and she does not invest or withdraw any money during this time. Calculate the value of her savings after 10 years.

3b. [3 marks] The rate of inflation during this 10 year period is 1.5% per year. Calculate the real value of her savings after 10 years.

4a. [2 marks] Adesh wants to model the cooling of a metal rod. He heats the rod and records its temperature as it cools.

Time, t (seconds)	0	30	60	90	120	150
Temperature, T ($^{\circ}\text{C}$)	75.6	62.2	53.3	47.4	42.3	38.5

He believes the temperature can be modeled by $T(t) = ae^{bt} + 25$, where $a, b \in \mathbb{R}$. Show that $\ln(T-25) = bt + \ln a$.

4b. [3 marks] Find the equation of the regression line of $\ln(T-25)$ on t .

4c. [3 marks] Hence find the value of a and of b .

4d. [2 marks] predict the temperature of the metal rod after 3 minutes.

5a. [1 mark] Consider the function $f(x) = \frac{3x+1}{x-2}$, $x \neq 2$. For the graph of f , write down the equation of the vertical asymptote.

5b. [2 marks] For the graph of f , find the equation of the horizontal asymptote.

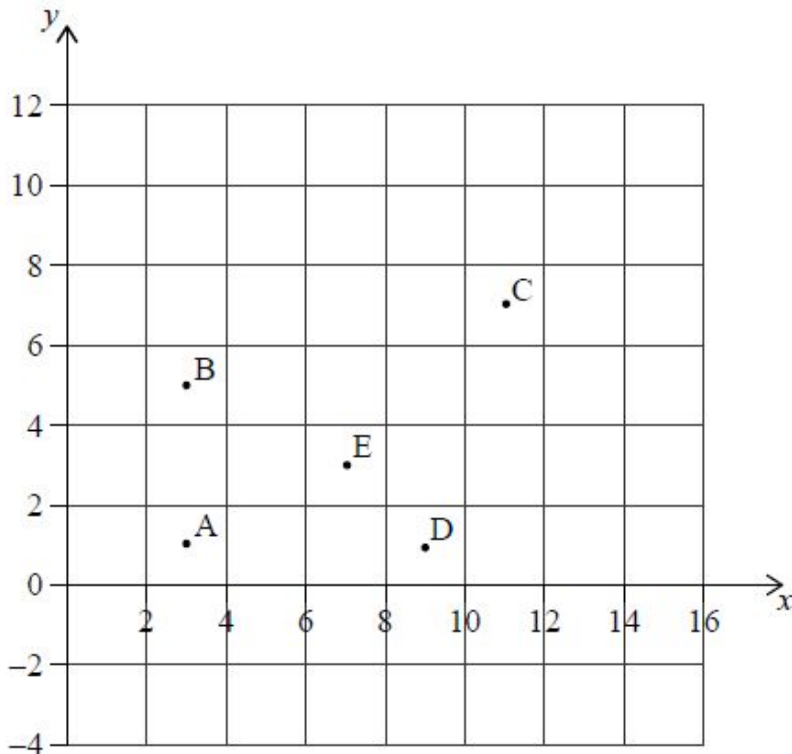
6a. [1 mark] The following diagram shows part of the graph of f with x -intercept $(5, 0)$ and y -intercept $(0, 8)$. Find the y -intercept of the graph of $f(x) + 3$.

6b. [2 marks] Find the y -intercept of the graph of $f(4x)$.

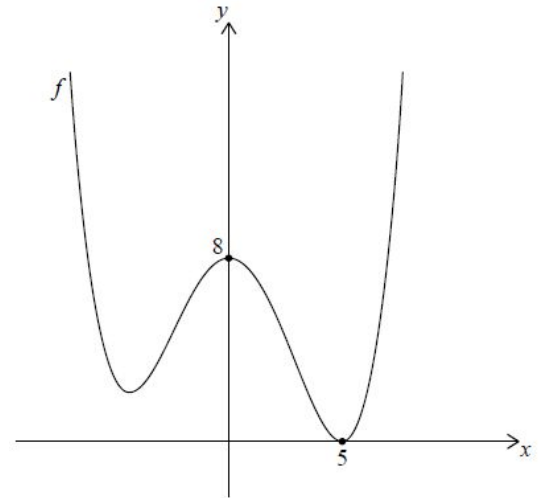
6c. [2 marks] Find the x -intercept of the graph of $f(2x)$.

6d. [2 marks] Describe the transformation $f(x + 1)$.

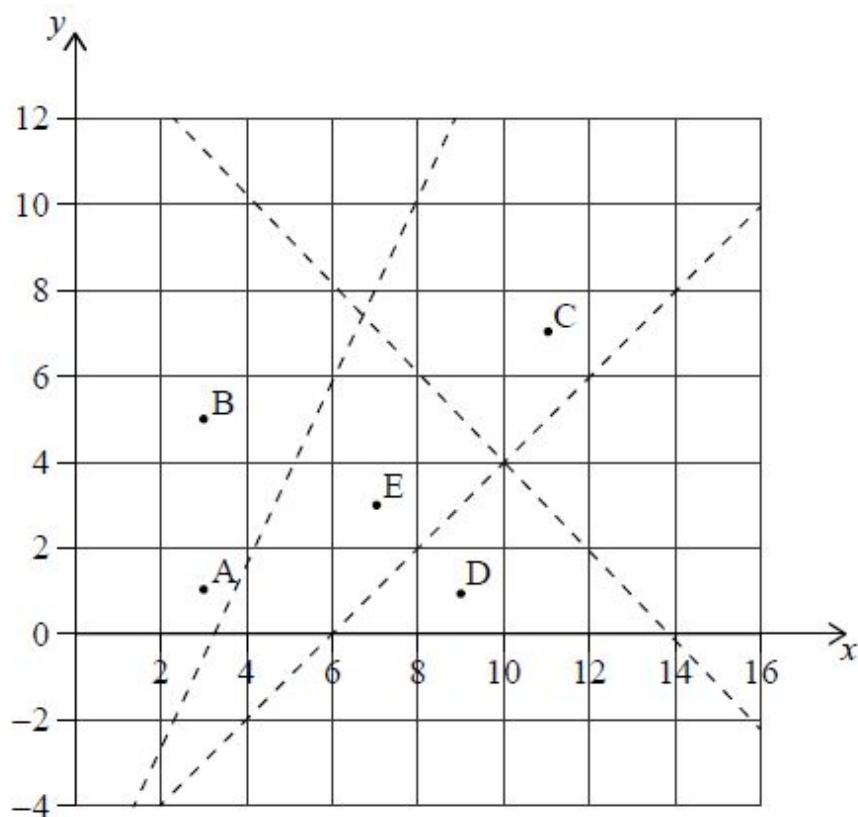
7a. [2 marks] Points A(3, 1), B(3, 5), C(11, 7), D(9, 1) and E(7, 3) represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes. Horizontal scale: 1 unit represents 1 km. Vertical scale: 1 unit represents 1 km.



Calculate the gradient of the line segment AE.



7b. [3 marks] The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



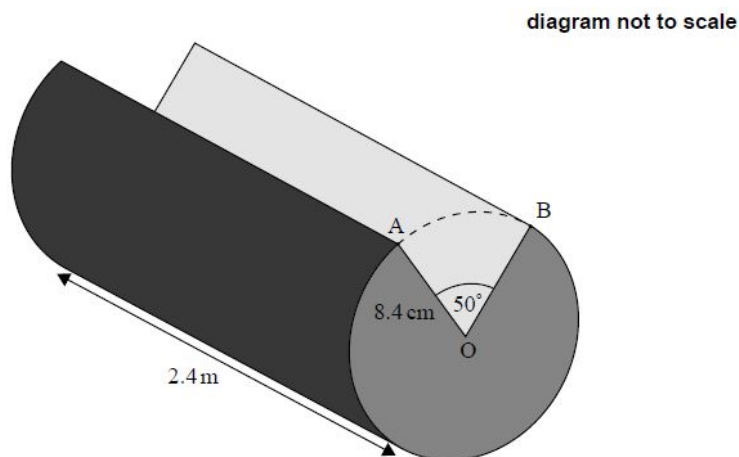
Find the equation of the line which would complete the Voronoi cell containing site E.
Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

7c. [1 mark] In the context of the question, explain the significance of the Voronoi cell containing site E.

8a. [1 mark] Helen is building a cabin using cylindrical logs of length 2.4 m and radius 8.4 cm. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.

Find 50° in radians.

8b. [4 marks] Find the volume of this log.



Markscheme

1a. [1 mark] ($d =$) - 250 **A1 [1 mark]**

1b. [2 marks] ($u_{16} =$) $6800 + (16-1)(-250)$ **M1** (¥)3050 **A1 [2 marks]**

1c. [3 marks] ($S_{16} =$) $(\frac{16}{2})(2 \times 6800 + (16-1)(-250)) \times 2$ **M1M1 Note:** Award **M1** for correct substitution into arithmetic series formula. Award **M1** for multiplication by 2 seen.

OR

($S_{16} =$) $(\frac{16}{2})(6800 + 3050) \times 2$ **M1M1 Note:** Award **M1** for correct substitution into arithmetic series formula. Award **M1** for multiplication by 2 seen. (¥)158 000 (157 600) **A1 [3 marks]**

2a. [2 marks] $10 \log_{10}(6.4 \times 10^{-3} \times 10^{12})$ (**M1**) = 98.1(dB) (98.06179...) **A1 [2 marks]**

2b. [2 marks] $112 = 10 \log_{10}(S \times 10^{12})$ (**M1**) 0.158 (W m^{-2}) (0.158489... (W m^{-2})) **A1 [2 marks]**

3a. [2 marks] $2400(1.04)^{10} = \$3552.59$ **M1A1 [2 marks]**

3b. [3 marks] real interest rate = $4 - 1.5 = 2.5\%$ **A1** $2400(1.025)^{10} = \$3072.20$ **M1A1 [3 marks]**

4a. [2 marks] $\ln(T-25) = \ln(ae^{bt})$ **M1** $\ln(T-25) = \ln a + \ln(e^{bt})$ **A1** $\ln(T-25) = bt + \ln a$
AG [2 marks]

4b. [3 marks] $\ln(T-25) = -0.00870t + 3.89$ **M1A1A1 [3 marks]**

4c. [3 marks] $b = -0.00870$ **A1** $a = e^{3.89\dots} = 49.1$ **M1A1 [3 marks]**

4d. [2 marks] $T(180) = 49.1e^{-0.00870(180)} + 25 = 35.2$ **M1A1 [2 marks]**

5a. [1 mark] $x = 2$ (must be an equation) **A1 N1 [1 mark]**

5b. [2 marks] valid approach (**M1**) eg $3 + \frac{7}{x-2}$, $x \rightarrow \infty$, $\frac{3x}{x}$, $\frac{3}{1}$, $\frac{3+\frac{1}{x}}{1-\frac{2}{x}}$, $\frac{3(x-2)+7}{x-2}$ $y = 3$ (must be an equation) **A1 N2 [2 marks]**

6a. [1 mark] y -intercept is 11 (accept (0, 11)) **A1 N1 [1 mark]**

6b. [2 marks] valid approach (**M1**) eg $f(4 \times 0) = f(0)$, recognizing stretch of $\frac{1}{4}$ in x -direction
 y -intercept is 8 (accept (0, 8)) **A1 N2 [2 marks]**

6c. [2 marks] x -intercept is $\frac{5}{2}$ (= 2.5) (accept ($\frac{5}{2}$, 0) or (2.5, 0)) **A2 N2 [2 marks]**

6d. [2 marks] correct name, correct magnitude **and** direction **A1A1 N2** eg name: translation,
(horizontal) shift (do not accept move) eg magnitude and direction: 1 unit to the left, (-1 0), horizontal
by -1 **[2 marks]**

7a. [2 marks] $\frac{3-1}{7-3}$ (**M1**) = 0.5 **A1 [2 marks]**

7b. [3 marks] $y-2 = -2(x-5)$ **(A1) (M1) Note:** Award **(A1)** for their -2 seen, award **(M1)** for the correct substitution of $(5, 2)$ and their normal gradient in equation of a line.

$2x + y - 12 = 0$ **A1 [3 marks]**

7c. [1 mark] every point in the cell is closer to E than any other snow shelter **A1 [1 mark]**

8a. [1 mark] $\frac{50 \times \pi}{180} = 0.873$ $(0.872664 \dots)$ **A1 [1 mark]**

8b. [4 marks] volume = $240 \left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664 \dots \right)$ **M1M1M1 Note:** Award **M1** $240 \times$ area, award **M1** for correctly substituting area sector formula, award **M1** for subtraction of the angles or their areas. = 45800 (= 45811.96071) **A1 [4 marks]**